Statistics 5401
October 17, 2005
On the following pages are charts that allow you (a) to get critical values for $\hat{\theta}_{1}=\frac{\hat{\lambda}_{1}}{1+\hat{\lambda}_{1}}$, where $\hat{\lambda}_{1}$ is the largest eigenvalue of $\mathbf{H}$ relative to $\mathbf{E}$ (largest eigenvalue of $\mathbf{E}^{-1} \mathbf{H}$ ), and (b) to test the significance of observed $\hat{\lambda}_{1}$.

Each chart goes with one value of $s=\min \left(p, f_{h}\right)$ for a particular value of $\alpha$. There are charts for $s=2,3,4$ and 5 and $\alpha=.05, .025$ and .01 .
Each chart contains graphs that are effectively double the width of the chart, with the curves at the lower left continuing (with a little overlap) the curves starting at the upper left.
The two scales below the X -axis represent values of $\hat{\theta}_{1}$. The upper scale, from 0 to .550 goes with the upper set of curves; the lower scale, from .500 to 1 , goes with the lower curve.
Each curve goes with one value of $m=-1 / 2,0,1,2,3, \ldots, 10$, where $m=\left(\left|p-f_{h}\right|-1\right) / 2$. Note that the two bottom curves in each group are for $m=-1 / 2$ and $m=0$ while the remaining curves are for $m=1,2, \ldots, 10$ stepping by 1. For intermediate values like $m=5 / 2$ you need to interpolate between the curves for $m=2$ and $m=3$.
The Y-axis represents $n$ from 5 to 1000 .

## Usage:

(a) To find a critical value:

Find where the curve for $m$ crosses the horizontal line corresponding to $n$. The horizontal position of the intersection is the critical value $\hat{\theta}_{1, \alpha}$ for $\hat{\theta}_{1}=\hat{\lambda}_{1} /\left(1+\hat{\lambda}_{1}\right)$.


You can compute a critical value $\hat{\lambda}_{1, \alpha}$ for $\hat{\lambda}_{1}$ as $\hat{\lambda}_{1, \alpha}=\frac{\hat{\theta}_{1, \alpha}}{1-\hat{\theta}_{1, \alpha}}$.
(b) To test significance of observed $\hat{\lambda}_{1}$.

Compute $\hat{\theta}_{1}=\hat{\lambda}_{1} /\left(1+\hat{\lambda}_{1}\right), s, m$ and $n$.
On chart for s, if point $\left(\hat{\theta}_{1}, n\right)$ is above line for $m$, reject $H_{0}$; otherwise you cannot reject. In the sketch, $\hat{\theta}_{1}$ is significantly large.

The charts are from Heck, D.L. Charts of Some Upper Percentage Points of the
Distribution of the Largest Characteristic Root, Ann.Math.Statist. 31 (1960) 625-642.

Chart I, $\mathrm{s}=2, \alpha=.01$


Chart II, $\mathrm{s}=2, \alpha=.025$


Chart III, $s=2, \alpha=.05$


Chart IV, $\mathrm{s}=3, \alpha=.01$


Chart V, s=3, $\alpha=.025$



Chart VII, $s=4, \alpha=.01$


Chart VIII, $s=4, \alpha=.025$


Chart IX, s = 4, $\alpha=.05$


Chart XI, $s=5, \alpha=.025$


Chart XII, s = 5, $\alpha=.05$


